Research illustrates importance of aligning teaching to standards

In 2019, the South Dakota Department of Education worked with REL (Regional Educational Laboratory) Central to research which South Dakota school districts, schools, or grade levels within schools are consistently realizing greater success in student math achievement and what strategies teachers are using.

47% of South Dakota students scored proficient or advanced on the 2019 statewide math assessment

Researchers conducted 36 interviews with teachers and administrators at elementary, middle, and high schools with significantly above average and significantly below average scores on the statewide math assessment and that tested 20 students or more. In addition, researchers focused on educators in schools with significantly higher math scores <u>and</u> above average populations of economically disadvantaged students.

Five primary response topics emerged:

- Alignment to standards
- Math instruction/pedagogy
- Support for effective instruction
- Students' experience in math
- Teacher beliefs about students

Schools with math scores significantly above average:

- Alignment to standards was discussed almost 100 more times
- Math instruction/pedagogy was discussed 84 more times

Schools with math scores significantly below average:

- Systems of intervention were discussed 41 more times

"I look at how students arrived at their answer and their proof and their justifications. That's where I put most of my emphasis when I assess." - Middle school educator

The research indicates that at schools with higher math scores on the statewide math assessment, teachers are teaching content standards to the required depth and intention and have best practices in place which support students in learning these standards.

At schools with lower math scores on the statewide math assessment, the research indicates that more emphasis is put on intervention than on teaching the grade level-aligned standards.
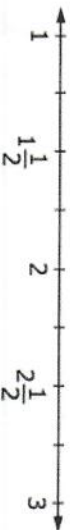
Moving forward:

SD Regional Math Circles (began 2021): educators step into the role of students by first tackling a challenging math task in groups to build their conceptual understanding, then engage in grade-specific math tasks to discuss planning, teaching, and standards progression

Foundational Mathematics course (began 2020): educators learn and discuss fundamental math progressions embedded in student understanding of number sense, how to assess student understanding, and best practices for lesson development and teaching

Accreditation: Department of Education supports school districts in preparation for demonstrating curriculum alignment to current state content standards to help ensure standards are taught to intended depth

Sharon Vestal, President, South Dakota Council of Teachers of Mathematics

Current standard	Proposed standard (s)	Sample Assessment Items & Comments about Proposed Standards and Assessment
3NF.A.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts (example: 1 part out of 4 equal parts is the same as 1/4); understand a fraction a/b as the quantity formed by a parts of size 1/b. (example:3/4 is the same as 3 one-fourths (1/4, 1/4, 1/4))	**3.F.2** Understand the denominator of a fraction represents the number of equal parts the whole is broken into. **3.F.3** Understand the numerator of a fraction represents the number of selected equal parts	**183391** Enter the fraction located at Point A on the number line. 0 A 1 *Source:* https://sampleitems.smarterbalanced.org/Item/200-183391. **Comments:** *The wording "the number of equal parts the whole is broken into" isn't well worded and doesn't sound professional. It seems unnecessary to break this standard into two standards as you would likely assess them in the same question so why not keep it as one standard.*
5MD.B.2 Make a line plot to display a data set. a. Use operations on fractions of a unit (1/2, 1/4, 1/8) for this grade to solve problems involving information presented in line plots. b. Use information from a line plot representing an unequal situation and redistribute whole or fractional parts to create an equal distribution. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally	**4.M.5** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8) and solve problems using the line plot. **5.M.2** Create and use a line plot to display a data set of measurements to solve real world problems involving information presented.	A scientist measures the width of ten different tree branches, **in inches**. [↗] [Delete ✕] The results, in inches, are 18, 24, 27, 30, 21, 18, 24, 30, 30 and 24. Complete the line plot to represent all of the results, in **feet,** by clicking above each tick mark to make an X appear. 1 1½ 2 2½ 3 *Source:* https://sampleitems.smarterbalanced.org/Item/200-183260 **Comment:** *It seems that a 5th grade standard was moved to 4th grade, but it is on the 5th grade Smarter Balanced practice test. This illustrates a lack of alignment.*

6RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

6.PR.2 Use ratio and rate reasoning to solve real-world and mathematical problems including making ratio tables, solving unit rate problems, and using percentages.

Fill in the missing values to complete the table.

The table shows the relationship between the amounts of ginger ale and fruit juice needed to make punch.

Ginger Ale (L)	Fruit Juice (oz)
	12
3	18
5	30
9	

Source: https://sampleitems.smarterbalanced.org/item/200-183499

Comment: *The wording in the original standard makes it very clear what the expectations are in terms of assessment. The proposed standard leaves things unclear as to what is really expected.*

7SP.B Draw informal comparative inferences about two populations. 3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, using quantitative measures of center (focusing on mean and median) and variability (interquartile range, mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable. 4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.	7.SP.1 Represent a set of data using a box plot, labeling quartiles, and identifying outliers. 7.SP.2 Select an appropriate measure(s) of center or variability and draw valid comparative inferences for two data sets. **6.SP.3** Represent numerical data on a number line, histogram, and box plot. **6.SP.4** Interpret a box plot to answer statistical questions about a data set.	**16309** III The box plots show a summary of the number of students present in Mr. Cooper's class and Mr. Parker's class during the last month. Determine whether each statement is true based on the box plots. Select True or False for each statement. (table below)

	True	False
The interquartile range of students present in Mr. Cooper's class is less than the interquartile range of students present in Mr. Parker's class.	☐	☐
The median number of students present in Mr. Parker's class is less than the median number of students present in Mr. Cooper's class.	☐	☐
The range of students present in Mr. Cooper's class is less than the range of students present in Mr. Parker's class.	☐	☐

Source: https://sampleitems.smarterbalanced.org/Item/200-16309

Comment: *The proposed standard eliminates comparison of visual displays and overall patterns. This assessment question is from the 7th grade Smarter Balanced, but it no longer aligns with the 7th grade standards. The 7th grade standards that might have fit this question are deleted now?*

High School Geometry
S.CP.A.4

Construct and interpret two-way frequency tables of data. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

Nothing similar to this standard is in the proposed standards

A1.SP.3 Summarize data from two categorical variables in a two-way frequency table; interpret relative frequencies in the context of the data, recognizing data trends and associations.

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27. On a trip, 2 sisters counted 1,430 vehicles. They divided the vehicles into categories: cars, trucks, and other. They noted the color of each as white, black, red, or other, as shown in the table. What is the probability that a randomly selected truck is black?

	White	Black	Red	Other	Total
Car	118	62	97	197	474
Truck	100	31	116	232	479
Other	86	85	94	212	477
Total	304	178	307	641	1,430

   A. $\frac{31}{178}$

   B. $\frac{31}{479}$

   C. $\frac{31}{1,430}$

   D. $\frac{479}{1,430}$

Source: https://www.act.org/content/dam/act/secured/documents/Preparing-for-the-ACT.pdf

Comment: *Two-way frequency tables often appear on standardized tests. So the fact that this standard was completely removed from the proposed standards is concerning. This particular standard is currently in the high school Geometry standards.*

High School Geometry
G.GPE.A.1

Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G.C.4 Write the equation of a circle, given the radius and center, where the center is at the origin or another point.

G.C.5 Identify the center and radius of a circle, given the equation of a circle, where the center is at the origin or another point.

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16. A circle in the standard (x,y) coordinate plane has its center at $(3,-4)$ and passes through $(0,0)$. Which of the following is an equation for that circle?

   F. $x^2 + y^2 = 5$

   G. $(x-3)^2 + (y+4)^2 = 25$

   H. $(x+3)^2 + (y-4)^2 = 7$

   J. $(x+3)^2 + (y-4)^2 = 25$

Source: https://www.act.org/content/dam/act/secured/documents/ACT-Test-Prep-ACT-Practice-Test-2-Form.pdf

Comment: *The above sample ACT item comes directly from the current standard and neither of the proposed standards get at what is needed to solve the problem because G.C.4 states that you will be given the radius of the circle. This ACT sample question doesn't give you the radius.*

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| High School Algebra II
F.BF.B.4.a.
Find inverse functions. a. Solve an equation for the independent variable of a function f that has an inverse function and write an expression for the inverse. | **A2.F.8** Write and graph the inverse of a given function; understand that the graph of an inverse function is a reflection of the function over the line $y=x$.

The standard below is misleading.
A2.F.7 Explain how restricting the domain of a function allows for the creation of its inverse. | 45. Given that $f(x)=\sqrt[3]{2x-1}$, which of the following expressions is the inverse function, $f^{-1}(x)$?

A. $\dfrac{1}{\sqrt[3]{2x-1}}$

B. $\sqrt[3]{\dfrac{x+1}{2}}$

C. $\dfrac{x^3+1}{2}$

D. $(2x-1)^3$

Source: https://www.act.org/content/dam/act/secured/documents/ACT-Test-Prep-ACT-Practice-Test-2-Form.pdf

Comments: Standard A2.F.7 says "explain how restricting the domain of a function allows for the creation of its inverse." This seems to imply that you MUST always restrict the domain for a function to have an inverse, but that isn't true. There are times when you have to restrict the domain so that the inverse is a function, but any equation has an inverse, it just isn't always a function. Also, proposed standard A2.F.8 says "write and graph the inverse of a given function," and to me, it isn't clear that writing the inverse involves a process. The current standard says "find inverse functions," which is phrased better because there is an algebraic process for finding the inverse. |
| **7.G.A.1**
Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. | **7.M.1** Calculate the scale factor, compute the actual lengths from the scale in a drawing, and reproduce a scale drawing using another scale. | 27. A scale model, where 1 coordinate unit represents 1 mile, is drawn in the standard (x,y) coordinate plane. Angelo's house is at $(4,-3)$, Ella's house is at $(4,7)$, Troy's house is at $(-2,7)$, and Yoko's house is at $(-2,-3)$. Which of the following is closest to the area, in square miles, of the rectangle whose vertices are the real locations of the 4 houses?

A. 32
B. 36
C. 60
D. 100

Source: https://www.act.org/content/dam/act/secured/documents/ACT-Test-Prep-ACT-Practice-Test-2-Form.pdf

Comments:
The word "scale" only appears in the proposed standards 5 times and it is all related to the 7th grade standard. In the current standards, the word "scale" appears 22 times, as early as grade 2, and in the context of measurement, graphing data, marking the appropriate scale on an axis, scale drawings, and scale factor. |

As you know, the state report card just came out within the last 2 weeks. When the math standards were revised, I looked at the percentage of students scoring at each level in mathematics in South Dakota for grades 4 and 8. I was able to update my data with the new report card. This information is below in Tables 1 and 2.

Table 1: SD Grade 4 Math Percentages of All Students at the Four Levels over the last Five Years

Grade 4	Level 1	Level 2	Level 3	Level 4
2020 - 2021	20.56	30.21	27.88	17.44
2021 - 2022	22.44	29.47	28.19	19.3
2022 - 2023	22.3	29.71	27.79	19.92
2023 - 2024	22.6	29.36	27.27	20.44
2024 - 2025	21.67	29.33	27.69	21.16

Table 2: SD Grade 8 Math Percentages of All Students at the Four Levels over the last Five Years

Grade 8	Level 1	Level 2	Level 3	Level 4
2020 - 2021	29.22	27.5	20.16	17.01
2021 - 2022	33.9	27.64	19.31	17.52
2022 - 2023	33.51	28.1	20.35	17.16
2023 - 2024	32.94	26.08	20.67	19.22
2024 - 2025	33.66	25.08	19.93	20.81

Now we know that we want students at Levels 3 and 4, so I added those percentages together, and they are shown in the table below. I am excited to report that for both grades 4 and 8, those percentages have increased just over 3.5% over the last five years. **Why are we rewriting the math standards?**

Table 3: Percentage of Grade 4 and Grade 8 SD students at Proficient or Advanced

Year	Grade 4 Proficient & Advanced	Grade 8 Proficient & Advanced
2020 - 2021	45.32	37.17
2021 - 2022	47.49	36.83
2022 - 2023	47.71	37.51
2023 - 2024	47.71	39.89
2024 - 2025	48.85	40.74

Percentage of Grade 4 and Grade 8 SD students at Proficient or Advanced

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